Exercises on Integration - MTH 161

Fall 2014

1. Estimate the area under the graph of $f(x) = 9 - x^2$ from $x = 0$ to $x = 3$ using 6 approximating rectangles and right endpoints.

2. Use the definition of **area under the curve** to express the area under $f(x) = \sqrt[4]{x}$ from 0 to 16. Don't evaluate the limit.

3. Express the limit as a definite integral on the given interval.

   a) $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \sin x_i \Delta x$, on $[0, \pi]$.

   b) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{x_i}{1 + x_i} \Delta x$, on $[1, 5]$.

4. Given that $\int_{-4}^{9} f(t)dt = 46$, what is $\int_{-4}^{9} f(x)dx$.

5. Evaluate the following integrals interpreting them in terms of areas.

   a) $\int_{-1}^{2} |x|dx$

   b) $\int_{-3}^{3}(1 - 2x)dx$

6. Evaluate by **definition** the integral $\int_0^2 (2 - x^2)dx$

7. If $\int_1^6 f(x)dx = 13$ and $\int_{-3}^{1} f(x)dx = 11$. Find $\int_{-3}^{6} f(x)dx$.

8. If $\int_3^{10} f(x)dx = 9$ and $\int_3^{10} g(x)dx = -3$, find $\int_3^{10}[2f(x) + 6g(x)]dx$.

9. Use the comparison property of integrals to show that

$$2 \le \int_0^2 \sqrt{1 + x^3}\, dx \le 6$$

10. Evaluate the following definite integrals:

    a) $\int_{-1}^{3} x^5 dx$.

    b) $\int_0^2 (6x^2 - 4x + 5)dx$

    c) $\int_0^2 x(2 + x^5)dx$

    d) $\int_{-2}^{-1} \left(4y^3 + \dfrac{2}{y^3}\right) dy$

    e) $\int_{\pi}^{2\pi} \csc^2 \theta d\theta$

    f) $\int_0^{\pi/4} \dfrac{1 + \cos^2 \theta}{\cos^2 \theta} d\theta$

11. Evaluate the following indefinite integrals:

a) $\int x\sqrt{x}dx$

b) $\int(\cos x - 2\sin x)dx$

c) $\int(1-t)(2+t^2)dt$

d) $\int \frac{\sin x}{1-\sin^2 x}dx$

e) $\int \frac{\sin 2x}{\sin x}dx$

12. Verify by differentiation that the formula is correct

    (a) $\int \frac{x}{\sqrt{x^2+1}}dx = \sqrt{x^2+1}+C$

    (b) $\int x\cos x dx = x\sin x + \cos x + C$.

13. Let $G(t)$ be the rate of growth of a population at time t. What is the integral that would give the increase in population between times $t = 2$ and $t = 16$?

14. A population of a certain town is increasing at a rate of $t^2 + 2t + 3$ people per year. Find the increase of population during the next 10 years. If the current population is 3000, what will be the population 10 years later.

15. State the Fundamental Theorem of Calculus.

16. Problem 1, Chapter 4.4 (pg 240)

17. Use the Fundamental theorem of Calculus to find the derivative of the following functions

    a) $g(x) = \int_{35}^{x} \sqrt{t^2+1}\, dt$

    b) $h(x) = \int_{0}^{x^4} \cos t\, dt$

    c) $r(x) = \int_{x}^{14} \frac{t}{t^2+1}\, dt$

    d) $p(x) = \int_{x}^{x^2} (y^2+2)\, dy$

18. Evaluate the following indefinite integrals

    a) $\int x\sin(x^2)\, dx$

    b) $\int \sec^2(2\theta)\, d\theta$

    c) $\int \frac{x}{(x^2+1)^3}\, dx$

    d) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

    e) $\int \sin^4 x\cos x\, dx$

    f) $\int \frac{\cos\left(\frac{1}{x}\right)}{x^2}\, dx$

    g) $\int x(2x+3)^6\, dx$

    h) $\int \sin\cos(\cos x)\, dx$

19. Evaluate the following indefinite integrals

a) $\int_0^1 \sqrt[3]{1+7x}\, dx$

b) $\int_0^1 (8x^3+3x^2)\, dx$

c) $\int_0^2 y^2\sqrt{1+y^3}\, dy$

d) $\int_0^{\frac{\pi}{4}} (1+\tan t)^3 \sec^2 t\, dt$

e) $\int_{-3}^3 x^4 \tan x\, dx$

f) $\int_{-1}^1 \dfrac{\sin x}{1+x^2}\, dx$

20. If $f^{'}$ is continuous on $[a,b]$, show that

$$2\int_a^b f(x)f^{'}(x)dx = [f(b)]^2 - [f(a)]^2$$

21. If f is continuous and $\int_0^9 f(x)dx = 4$, find $\int_0^3 uf(u^2)du$

22. Find the average value of the function on the given interval

a) $f(x) = 4x - x^2,\ [0,4]$

b) $h(t) = \dfrac{3}{(1+t)^2},\ [1,6]$